SNFactory-SNIFS/02-002

SNIFS optical parametrization

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April 26, 2002

1 Distorsion of the spectra pattern

The basic geometric parameters of SNIFS have been computed to avoid any overlapping of the spectra. However, these computations have been led assuming a perfect optics, free of any distorsion. There is two sources of distorsion: the geometric distorsion, which is one of the Seidel aberration, and the lateral color which shift lateraly the instrument focus, leading to a chromatic distorsion of the spectra. We describe here the mathematic parametrization of these two effects.

1.1 Geometric distorsion

In the gaussian aproximation, a lens of focal f transforms a direction θ in the object space into a height y in the image focal plane following the classical formula:

$$y = f \tan theta$$

If the inclination of the beam becomes too large, terms of higher orders appear:

$$y = f \tan\theta(1 + E_2 \tan^2\theta + E_4 \tan^4\theta + ...)$$

This parametrization is useful when the lens is used as a camera (transforming a direction into a position). When the lens is a collimator (transforming a height in the object focal plane into a direction) the parametrization will be:

$$\tan\theta = y(1 + \epsilon_2 y^2 + \epsilon_4 y^4 + ...)$$

This two formula will be used to parametrize the distorsion of SNIFS collimator and camera. For a given lens, Zemax provides the curve $D(y_r)$, where $y_r = \tan(\theta)$ is the undistorted height in the image plane and D is defined by:

$$D = \frac{y - y_r}{y_r}$$

y beeing the chief ray impact in the focal plane. The focal length f is also provided by Zemax, the camera distorsion parameters $E_2, E_4, ...$ can be directly determined by fitting the model to the Zemax curve. Indeed, the distorsion equation of the camera can be rewriten:

$$y = y_r\left(1 + E_2\left(\frac{y_r}{f_{cam}}\right)^2 + E_4\left(\frac{y_r}{f_{cam}}\right)^4 + ...\right)$$

The distorsion D is then:

$$D = E_2\left(\frac{y_r}{f_{cam}}\right)^2 + E_4\left(\frac{y_r}{f_{cam}}\right)^4 + ...$$

and we just have to fit this model to the Zemax curves to obtain the distorsion paramaters. For the collimator, we want $\theta(y_{obj})$. The distorsion curve is computed for an optical design made of the collimator plus an ideal lens (free of any distorsion or aberration) of focal length f_{cam}. Zemax provides the curve $D(y_r)$ for this design. In this case we have $y_r = y_{obj}\frac{f_{cam}}{f_{coll}}$ and $y = f_{cam}\tan\theta$. The Zemax curve :

$$y = y_r(1 + D)$$

transforms in:

$$\tan\theta = \frac{y_{obj}}{f_{coll}}(1 + D)$$

Thus, we just have to fit the model to the distorsion to find the collimator distorsion parameters $\epsilon_2, \epsilon_4, ...$

As shown in figure 1, a development to the second order is enough for the distorsion of each element of the spectrograph.

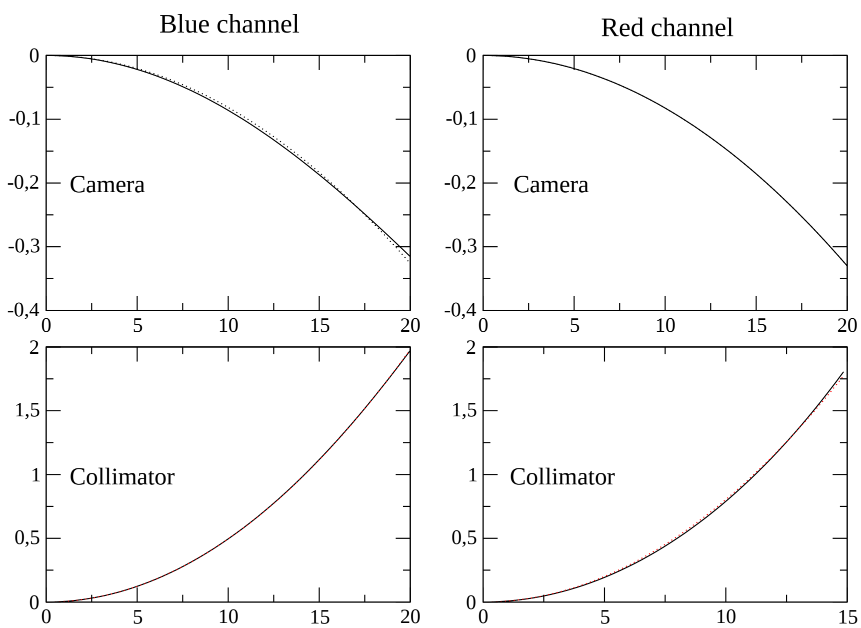

Figure 1: Fit of the distorsion curves for each SNIFS element. In each case a second order distorsion has been fitted. The model is almost undiscernable from the data.

1.2 Lateral color

The lateral color aberration is a radial shift of the lens focus. Zemax provides for each wavelength the value of the focus shift in microns to the focus at a reference wavelength as a function of relative image heigth (defined as the ratio of the current image heigth to the maximum one). Figure 2 displays these curves for each SNIFS component. These curves are almost linear, and a simple model is fitted to each of them:

$$C(\lambda) = a(\lambda)u$$

where $u = \frac{y}{y_{max}}$. The lateral color of each SNIFS lens is then parametrized by the wavelength dependency of the coefficient $a(\lambda)$. This coefficient is fitted by a cubic curve, the fit are shown in figure 3.

To separate the camera and collimator contribution, the same optical designs than the ones used for the distorsion computations are used in Zemax. This provides to sets of coefficients: $a_{cam}(\lambda)$ and $a_{coll}(\lambda)$. For the camera, one have:

$$u = \frac{f_{cam}\tan\theta}{y_max}$$

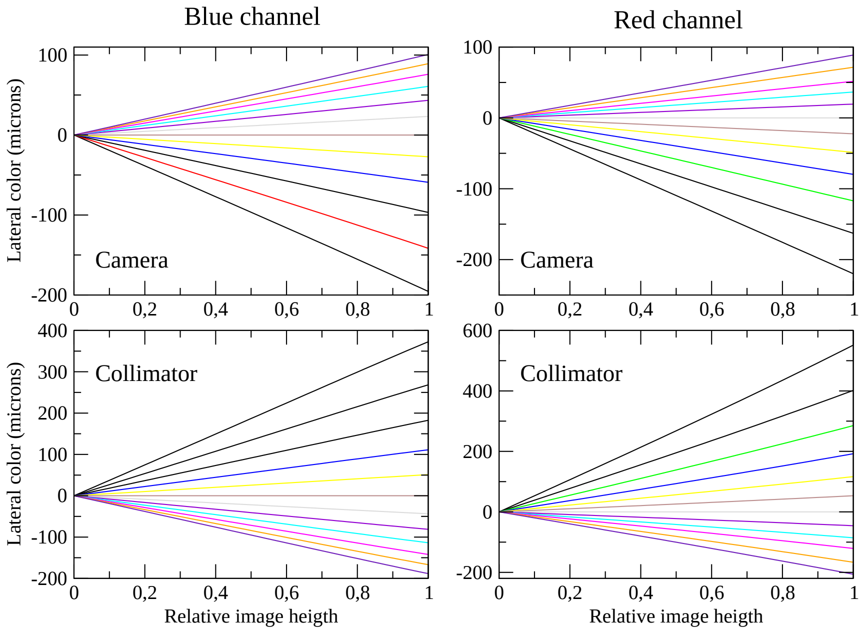

Figure 2: Lateral color for each SNIFS element. Different colors correspond to different wavelength

and thus

$$\Delta y = a_{cam}(\lambda)\frac{f_{cam}\tan\theta}{y_{max}}$$

And for the collimator:

$$u = \frac{f_{cam}}{f_{coll}}\frac{y}{y_{max}}$$

and

$$\Delta y = a_{coll}(\lambda)\frac{y}{f_{coll}}$$

1.3 Actual values of the distorsion coefficients

The complete distorsion parametrization for the collimator and camera are the following:

1. camera:

$$y = f_{cam}\tan\theta)(1 + e_{cam}\tan^2\theta + a_{cam}(\lambda)\tan\theta)$$

   where:

$$a_{cam}(\lambda) = \alpha_0^{cam} + \alpha_1^{cam}(\lambda - \lambda_{ref}) + \alpha_2^{cam}(\lambda - \lambda_{ref})^2 + \alpha_3^{cam}(\lambda - \lambda_{ref})^3$$

2. collimator:

$$\tan\theta = \frac{y}{f_{coll}}(1 + e_{coll}y^2 + a_{coll}y)$$

   where:

$$a_{coll}(\lambda) = \alpha_0^{coll} + \alpha_1^{coll}(\lambda - \lambda_{ref})\alpha_2^{coll}(\lambda - \lambda_{ref})^2 + \alpha_3^{coll}(\lambda - \lambda_{ref})^3$$

Table 1.3 provides the values of each coefficient for the SNIFS model where the distance are in millimeter.

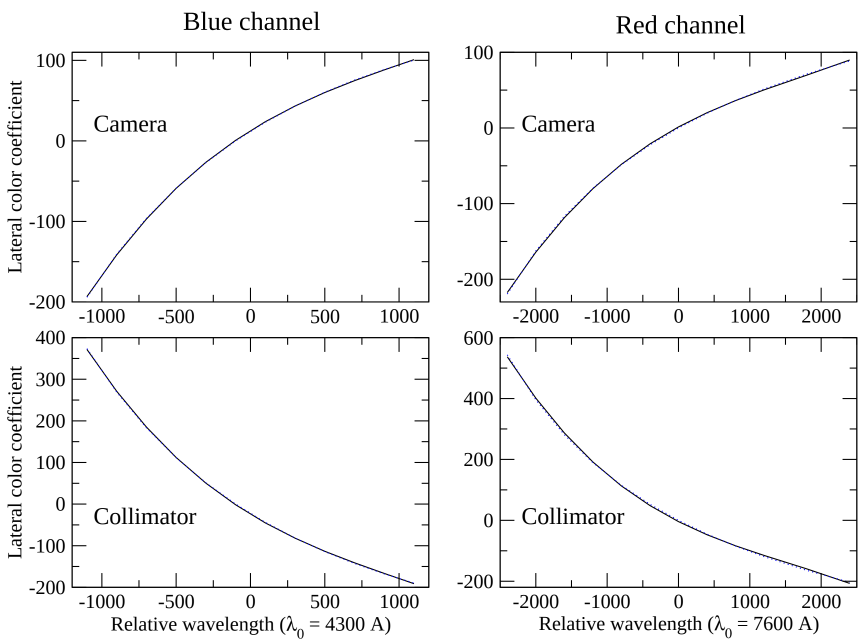

Figure 3: Fit of the lateral color coefficients $a(\lambda)$ of each SNIFS element by a cubic curve

	Blue channel		Red channel	
	Collimator	Camera	Collimator	Camera
f	133.476	179.869	169.549	228.014
e	1.6	-0.286	2.017	-0.283
λ_{ref}	4300Å		7600Å	
α_0	$-1.218810.10^{-3}$	$6.378.10^{-3}$	$-7.9345.10^{-5}$	$7.043.10^{-5}$
α_1	$-1.08715.10^{-5}$	$5.57575.10^{-6}$	$-3.17765.10^{-6}$	$2.75485.10^{-6}$
α_2	$4.731.10^{-9}$	$-2.43585.10^{-9}$	$6.6145.10^{-10}$	$-5.851.10^{-10}$
α_3	$-1.5868.10^{-12}$	$7.6595.10^{-13}$	$-1.3108.10^{-13}$	$1.1589.10^{-13}$

Table 1: Actual value of the SNIFS optical parameters

2 Energy in the different grating orders

The transmission of a perfect transmission grating (neglecting any polarization effect) is given by the product of the grating function with the diffraction pattern of one groove of the grating:

$$I = \frac{\sin^2(N\Psi)}{\sin^2\Psi}\frac{\sin^2\Theta}{\Theta^2} \tag{1}$$

where

$$\Theta = \frac{\pi a(n\sin i - \sin r)}{\lambda}, \quad \Psi = \frac{\pi b(n\sin i - \sin r)}{\lambda} \tag{2}$$

with n being the index of the grating substrate, i the incidence angle, r the refracted angle, a the size of the grooves, b the grating step and N the number of grooves of the grating. The maximum of energy is sent in the 0 order, and the gratings are thus usually blazed to a given angle to shift the diffraction pattern of the grooves with respect to the grating function. If the blaze angle is γ, Θ is rewriten:

$$\Theta = \frac{\pi b\cos\gamma(n\sin i - \sin r)}{\lambda} \tag{3}$$

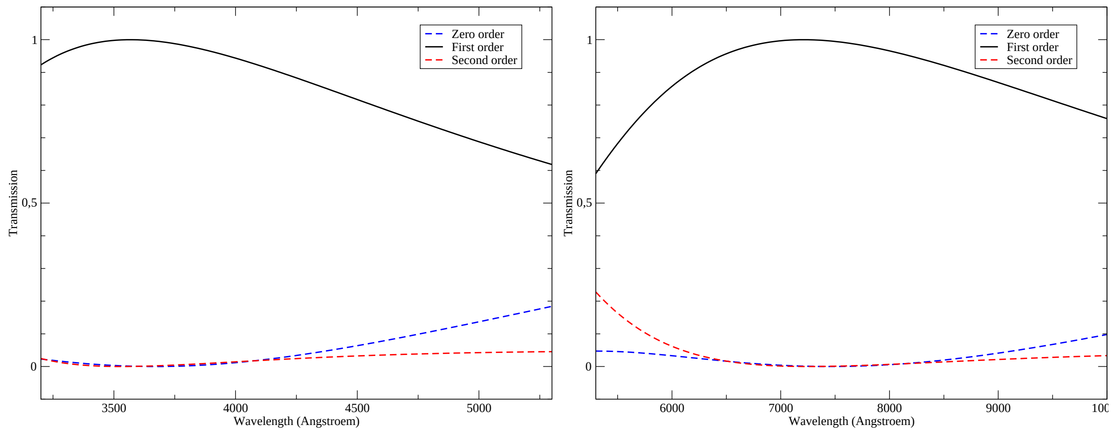

Figure 4: Gratings transmission for the orders 0,1,2. Left: Blue channel. Right: Red channel

Note that i and r are no more the angle from the grating normal but to the grating groove normal, these two direction being no more aligned. The function $\frac{\sin^2\Theta}{\Theta^2}$ is then called the blaze function, and provides the theoretical transmission curve of the grating. If α and β are the incident and refracted angles from the grating normal, one have the following relations:

$$m\rho\lambda = n\sin\alpha - \sin\beta \tag{4}$$

where m is the diffration order and $\rho = 1/b$ the lines density of the grating.

$$i = \alpha - \gamma,\ r = \beta - \gamma \tag{5}$$

The transmission curves for the orders 0,1,2 are displayed in figure 4